Journal of Sound and Vibration (2000) **237**(1), 163–172
doi:10.1006/jsvi.2000.2973, available online at http://www.idealibrary.com on IDEAL®

LETTERS TO THE EDITOR

A NOTE ON SOUND RADIATION FROM A CONFINED SOURCE

JIE PAN

Department of Mechanical and Materials Engineering, The University of Western Australia, Nedlands, WA 6907, Australia. E-mail: pan@mech.uwa.edu.au

(*Received* 26 *August* 1999, *and in final form* 12 *January* 2000)

This note presents an expression of radiated sound pressure from a confined source in terms of the spherical harmonic functions. In the expression, the spherical harmonic functions are interpreted as the radiating modes and their superposition results in various radiation patterns. The orthonormal property of the radiating modes also allows a general expression of the multipole radiated sound power.

1. INTRODUCTION

The sound radiation from a confined source is a textbook subject. The properties of multiple radiation in the far field are also well known, and often illustrated by the sound radiation of a point source and superposition of multiple point sources. A general expression of the sound radiation from a confined source is available [1–3] in terms of the spatial derivatives of the free-field Green's function

$$p(\mathbf{r}, t) = \frac{Q_M}{4\pi r} \mathrm{e}^{\mathrm{j}\omega t - \mathrm{j}kr} - \mathbf{Q}_D \cdot \nabla\left(\frac{\mathrm{e}^{\mathrm{j}\omega t - \mathrm{j}kr}}{4\pi r}\right) + \boldsymbol{Q}_Q : \left[\nabla\left(\frac{\mathrm{e}^{\mathrm{j}\omega t - \mathrm{j}kr}}{4\pi r}\right)\nabla\right] + \cdots \tag{1}$$

for $r \gg r_0$. The monopole source strength and dipole strength vectors are, respectively,

$$Q_M = \mathrm{j}\rho_0\omega \int_{V_0} q_v(\mathbf{r}_0)\,\mathrm{d}V_0, \qquad \mathbf{Q}_D = \mathrm{j}\rho_0\omega \int_{V_0} r_0 q_v(\mathbf{r}_0)\,\mathrm{d}V_0. \tag{2, 3}$$

The xyth component of the quadrupole strength tensor $\boldsymbol{Q}_Q$ is

$$Q_{Qxy} = \mathrm{j}\rho_0\omega \int_{V_0} x_0 y_0 q_v(\mathbf{r}_0)\,\mathrm{d}V_0. \tag{4}$$

While the calculation of the radiated sound pressure from the volume velocity density $q_v(\mathbf{r}_0)$ of the source is possible, the directivity and basic components of the radiating sound field by equation (1) are not explicit. On the other hand, orthonormal functions of the elevation and azimuth angles (θ, ϕ), such as spherical harmonics functions, have been used to express the multipole expansion of the static potential field from a confined electrical charge distribution [4]. The 2^l multipole of the static field has an amplitude relating to the observation point r by $1/r^{l+1}$, where $l = 0, 1, 2, \ldots$. In acoustics, Morse and Ingard [1] have used spherical harmonics and spherical Hankel functions to express the multipole field of sound radiation from vibrating spheres. Their expression is not suitable for a distributed volume velocity because the spherical Hankel functions are singular at the origin of the co-ordinates.

Nevertheless, the sound radiation from the spheres can be decomposed into a few basic radiating modes described by the spherical harmonics. As a result, the discussion of the radiating field and power may be unified in terms of the basic radiating modes.

In this note, an expansion of the radiating sound from a confined source is presented by the spherical harmonics. The singularity problem at the origin is avoided as a new expansion method has been used. The concept of radiating modes is illustrated by examples of multipole sound radiation and the calculation of the multipole radiating power is demonstrated.

2. RADIATED SOUND PRESSURE

The sound pressure in free field is obtained using the Green's function technique. The steady state Green's equation and free-field Green's function leads to the expression of the sound pressure in free space generated by the volume velocity density of the sound source [1] as

$$p(\mathbf{r}, t) = \mathrm{j}\rho_0\omega \int \frac{q_v(\mathbf{r}_0)}{4\pi|\mathbf{r} - \mathbf{r}_0|} \mathrm{e}^{\mathrm{j}\omega t - \mathrm{j}k|\mathbf{r} - \mathbf{r}_0|} \, \mathrm{d}V_0. \tag{5}$$

For a confined source distribution $kr_0 \ll 1$, the sound field generated can be divided into three regions in terms of the ratio between the observation location r and the wavelength λ of the sound wave:

(a) near field: $r/\lambda \ll 1$,
(b) middle field: $r/\lambda \sim 1$,
(c) far field: $r/\lambda \gg 1$.

In the near-field region ($kr \ll 1$), the sound pressure distribution as a function of distance r is highly dependent upon the distribution of sound source. Not only by the $1/r$ terms, the sound pressure is also contributed by higher order terms such as $1/r^{l+1}(1/r^{l+1} l = 1, 2, 3, \ldots)$. In the farfield region ($kr \gg 1$) however, the condition of $r \gg r_0$ allows the following approximation:

$$|\mathbf{r} - \mathbf{r}_0| \cong r - r_0 \cos \Theta \tag{6}$$

for the exponential term in equation (5), where

$$\cos \Theta = \cos\theta \cos\theta_0 + \sin\theta \sin\theta_0 \cos(\phi - \phi_0) \tag{7}$$

and (θ_0, ϕ_0) are the elevation and azimuth angles of the source position vector $\mathbf{r}_0$. As a result the farfield sound pressure can be expressed as

$$p(\mathbf{r}, t) = \mathrm{j}\rho_0\omega \frac{\mathrm{e}^{\mathrm{j}\omega t - \mathrm{j}kr}}{4\pi r} \int q_v(\mathbf{r}_0) \mathrm{e}^{\mathrm{j}kr_0 \cos\Theta} \, \mathrm{d}V_0 \tag{8}$$

which is the typical radiating field with magnitude decaying as a function of r^{-1}.

3. MULTIPLE DESCRIPTION OF RADIATING SOUND FIELD

The farfield sound radiation from a confined source is then Taylor-expanded in terms of the non-dimensional size of the sound source kr_0:

$$p(\mathbf{r}, t) = j\rho_0\omega \frac{\mathrm{e}^{\mathrm{j}\omega t - \mathrm{j}kr}}{4\pi r} \sum_{l=0}^{\infty} \frac{1}{l!} \int q_v(\mathbf{r}_0)(\,\mathrm{j}kr_0 \cos\Theta)^l \,\mathrm{d}V_0. \tag{9}$$

To show the directivity property of $(\cos\Theta)^l$, the relationship between Legendre polynomials $P_l(\cos\Theta)$ and $(\cos\Theta)^l$ is investigated. According to the definition of Legendre polynomials, they are related to $x = \cos\Theta$ by

$$\begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_l \end{bmatrix} = \begin{bmatrix} c_{00} & & & & & \\ 0 & c_{11} & & \mathbf{0} & & \\ c_{20} & 0 & c_{22} & & & \\ 0 & c_{31} & 0 & c_{33} & & \\ & & & & \ddots & \\ & & & & & c_{ll} \end{bmatrix} \begin{bmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \\ \vdots \\ x^l \end{bmatrix}, \tag{10}$$

where

$$c_{nm} = \begin{cases} \dfrac{(-1)^{s_{nm}}(2n - 2s_{nm})!}{2^n s_{nm}!(n - s_{nm})!(n - 2s_{nm})!} & \begin{pmatrix} n \\ m \end{pmatrix} \in \text{odd}, \begin{pmatrix} n \\ m \end{pmatrix} \in \text{even}, \\ 0 & \begin{pmatrix} n \\ m \end{pmatrix} \in \begin{pmatrix} \text{odd} \\ \text{even} \end{pmatrix}, \begin{pmatrix} n \\ m \end{pmatrix} \in \begin{pmatrix} \text{even} \\ \text{odd} \end{pmatrix} \text{ or } m > n \end{cases}, \tag{11}$$

$s_{nm} = [n/2] - [m/2]$ and [ ] represents the nearest integer in the bracket.

Inverting the coefficient matrices in equation (10) gives rise to

$$\begin{bmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \\ \cdot \\ x^l \end{bmatrix} = \begin{bmatrix} D_{00} & & & & & \\ 0 & D_{11} & & \mathbf{0} & & \\ D_{20} & 0 & D_{22} & & & \\ 0 & D_{31} & 0 & D_{33} & & \\ & & & & \cdot & \\ & & & & & D_{ll} \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ \cdot \\ P_l \end{bmatrix}, \tag{12}$$

where

$$D_{ij} = \begin{cases} \dfrac{1}{c_{ii}} & i = j \\ -\dfrac{1}{c_{ii}} \displaystyle\sum_{k=j}^{i-1} c_{ik} D_{kj} & i \neq j, \begin{pmatrix} i \\ j \end{pmatrix} \in \text{odd}, \begin{pmatrix} i \\ j \end{pmatrix} \in \text{even}, \\ 0 & \begin{pmatrix} i \\ j \end{pmatrix} \in \begin{pmatrix} \text{odd} \\ \text{even} \end{pmatrix}, \begin{pmatrix} i \\ j \end{pmatrix} \in \begin{pmatrix} \text{even} \\ \text{odd} \end{pmatrix} \text{ or } j > i \end{cases}, \tag{13}$$

Substituting equation (12) into equation (9) and collecting the coefficients of P_l, the farfield sound pressure becomes

$$p(\mathbf{r}, t) = j\rho_0\omega \frac{\mathrm{e}^{\mathrm{j}\omega t - \mathrm{j}kr}}{4\pi r} \sum_{l=0}^{\infty} \int q_v(\mathbf{r}_0) a_l P_l(\cos\Theta)\,\mathrm{d}V_0. \tag{14}$$

where

$$a_l = \sum_{n=0}^{\infty} \frac{(jkr_0)^n}{n!} D_{nl}. \tag{15}$$

For confined source $kr_0 \ll 1$, the a_l are approximated by the first term in equation (15):

$$a_l \cong (\mathrm{j}kr_0)^l \frac{2^l l!}{(2l)!}. \tag{16}$$

Taking the monopole, dipole and quadrupole radiation ($l = 0, 1, 2$) as an example, the a_l are approximated as

$$a_0 \cong 1, \qquad a_1 \cong \mathrm{j}kr_0, \qquad a_2 \cong \tfrac{1}{3}(\mathrm{j}kr_0)^2. \tag{17}$$

Finally, using the spherical harmonic functions for the Legendre polynomials, we obtained the multipole expression of the radiating sound field

$$p(\mathbf{r}, t) = j\rho_0\omega \frac{\mathrm{e}^{\mathrm{j}\omega t - \mathrm{j}kr}}{r} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} q_{lm} Y_{lm}(\theta, \phi), \tag{18}$$

where the coefficients of the (l, m)th order radiating modes are

$$q_{lm} = \frac{(-1)^m}{2l+1} \int q_v(\mathbf{r}_0) a_l Y^*_{lm}(\theta_0, \phi_0)\,\mathrm{d}V_0. \tag{19}$$

The spherical harmonic function is defined as

$$Y_{lm}(\theta, \phi) = \sqrt{\frac{(2l+1)}{4\pi} \frac{(l-m)!}{(l-m)!}}\, P_l^m(\cos\theta)\mathrm{e}^{\mathrm{j}m\phi}, \tag{20}$$

where the $P_l^m(\cos\theta)$ are the associated Legendre polynomials

$$P_l^m(x) = \frac{(-1)^m}{2^l l!} (1 - x^2)^{m/2} \frac{\mathrm{d}^{l+m}}{\mathrm{d}x^{l+m}} (x^2 - 1)^l \tag{21}$$

with $x = \cos\theta$. When m is a negative number, the following relationship is used:

$$Y_{l-m}(\theta, \phi) = (-1)^m Y^*_{lm}(\theta, \phi). \tag{22}$$

In equation (18), the monopole, dipole and quadrupole sound radiation are, respectively, represented by the radiation modes described by the spherical harmonic functions when $l = 0$ ($m = 0$), $l = 1$ ($m = -1, 0, 1$), and $l = 2$ ($m = -2, -1, 0, 1, 2$). Table 1 shows the corresponding radiating modes and their directivities. Examples of sound radiation from point sources are used to illustrate the use of the radiating modes.

TABLE 1

Spherical harmonic functions $Y_{lm}(\theta, \phi)$ and their directivities $|Y_{lm}(\theta, \phi)|^2$

$l=0$	$Y_{00}(\theta, \phi) = \dfrac{1}{\sqrt{4\pi}}$	
	$Y_{11}(\theta, \phi) = -\sqrt{\dfrac{3}{8\pi}}\sin\theta e^{j\phi}$	
$l=1$		
	$Y_{10}(\theta, \phi) = \sqrt{\dfrac{3}{4\pi}}\cos\theta$	
	$Y_{22}(\theta, \phi) = \dfrac{1}{4}\sqrt{\dfrac{15}{2\pi}}\sin^2\theta e^{j2\phi}$	
$l=2$	$Y_{21}(\theta, \phi) = -\sqrt{\dfrac{15}{8\pi}}\sin\theta\cos\theta e^{j\phi}$	
	$Y_{20}(\theta, \phi) = \sqrt{\dfrac{5}{4\pi}}\left(\dfrac{3}{2}\cos^2\theta - \dfrac{1}{2}\right)$	

3.1. MONOPOLE SOUND RADIATION

The volume velocity density of a point source with volume velocity Q at $\mathbf{r}_0 = 0$ is

$$q_v(\mathbf{r}_0) = Q\delta(\mathbf{r}_0). \tag{23}$$

With such a source distribution, the non-zero solution of equation (19) is

$$q_{00} = \frac{Q}{\sqrt{4\pi}} \tag{24}$$

which gives rise to the radiated monopole sound pressure

$$p(\mathbf{r}, t) = \frac{j\rho_0\omega Q}{4\pi r}e^{j\omega r - jkr}, \tag{25}$$

$j\rho_0\omega Q$ is also called the strength of the point source.

3.2. DIPOLE SOUND RADIATION

Figure 1 shows a distribution of two point sources with equal strength but opposite phase. They are located along the Z-axis at distance of d. The volume velocity density of the sources is

$$q_v(\mathbf{r}_0) = Q\delta\left(\mathbf{r}_0 - \frac{d}{2}\hat{z}\right) - Q\delta\left(\mathbf{r}_0 + \frac{d}{2}\hat{z}\right). \tag{26}$$

The non-zero term of equation (19) is

$$q_{10} = \frac{\mathrm{j}Qkd}{3}\sqrt{\frac{3}{4\pi}}. \tag{27}$$

Thus, the radiated sound pressure has the expression of

$$p(\mathbf{r}, t) = \mathrm{j}\rho_0\omega\frac{\mathrm{e}^{\mathrm{j}\omega t - \mathrm{j}kr}}{r}q_{10}Y_{10}(\theta, \phi) = -\rho_0\omega\frac{Qkd}{4\pi r}\cos\theta\,\mathrm{e}^{\mathrm{j}\omega t - \mathrm{j}kr} \tag{28}$$

which has the directivity of a dipole.

If the source distribution is along the Y-axis as shown in Figure 2, the volume velocity density is

$$q_v(\mathbf{r}_0) = Q\delta\left(\mathbf{r}_0 - \frac{d}{2}\hat{y}\right) - Q\delta\left(\mathbf{r}_0 + \frac{d}{2}\hat{y}\right). \tag{29}$$

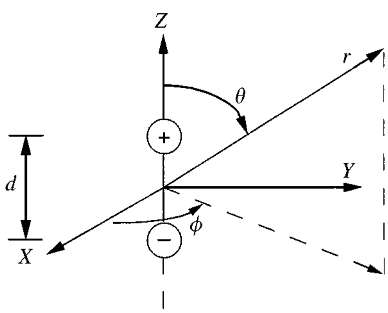

Figure 1. Spherical co-ordinates and a dipole sound source distribution.

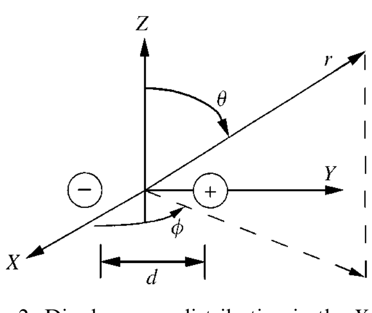

Figure 2. Dipole source distribution in the Y-axis.

The corresponding non-zero terms of equation (19) are

$$q_{11} = \frac{1}{3} Qkd \sqrt{\frac{3}{8\pi}} \qquad q_{1-1} = q_{11}. \tag{30a, b}$$

Thus, the resultant sound field is

$$p(\mathbf{r}, t) = \mathrm{j}\rho_0\omega \frac{\mathrm{e}^{\mathrm{j}\omega t - \mathrm{j}kr}}{r} [q_{11} Y_{11}(\theta, \phi) + q_{1-1} Y_{1-1}(\theta, \phi)]$$

$$= -\,\mathrm{j}\rho_0\omega \frac{Qkd}{4\pi r} \sin\theta \cos\phi\, \mathrm{e}^{\mathrm{j}\omega t - \mathrm{j}kr} \tag{31}$$

which also shows a dipole directivity. Note that the dumbbell radiation directivity in the Y-axis is due to the superposition of two doughnut-shaped radiating modes ($Y_{11}(\theta, \phi)$ and $Y_{1-1}(\theta, \phi)$).

3.3. QUADRUPOLE SOUND RADIATION

Both longitudinal (Figure 3(a)) and lateral (Figure 3(b)) quadrupoles are also special cases of source distribution according to equation (19).

The source distributions of the longitudinal quadrupole is

$$q_v(\mathbf{r}_0) = Q\delta(\mathbf{r}_0 - \mathrm{d}\hat{y}) - 2Q\delta(\mathbf{r}_0) + Q\delta(\mathbf{r}_0 + \mathrm{d}\hat{y}) \tag{32}$$

and that of the lateral quadrupole is

$$q_v(\mathbf{r}_0) = Q\delta\left(\mathbf{r}_0 - \frac{d}{2}\hat{y} - \frac{d}{2}\hat{z}\right) - Q\delta\left(\mathbf{r}_0 + \frac{d}{2}\hat{y} - \frac{d}{2}\hat{z}\right)$$

$$+ Q\delta\left(\mathbf{r}_0 + \frac{d}{2}\hat{y} + \frac{d}{2}\hat{z}\right) - Q\delta\left(\mathbf{r}_0 - \frac{d}{2}\hat{y} + \frac{d}{2}\hat{z}\right) \tag{33}$$

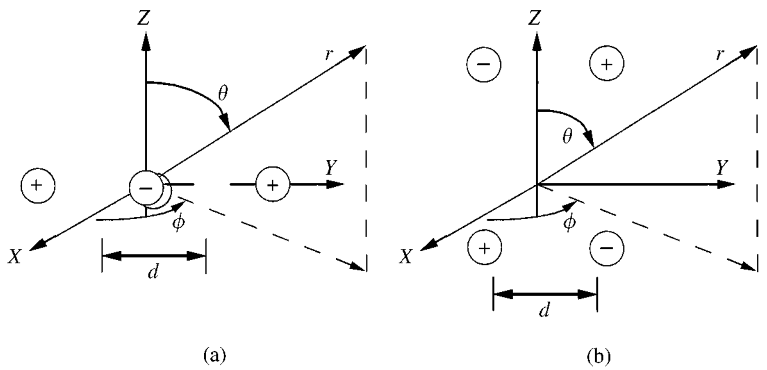

Figure 3. (a) A longitudinal quadrupole and (b) a lateral quadrupole.

For longitudinal quadrupole, the non-zero q_{lm} are

$$q_{22} = -\frac{1}{30} Q(kd)^2 \sqrt{\frac{15}{2\pi}}, \qquad q_{2-2} = q_{22}, \qquad q_{20} = -\frac{1}{15} Q(kd)^2 \sqrt{\frac{5}{4\pi}}. \quad \text{(34a–c)}$$

Thus, the radiated sound pressure is

$$p(\mathbf{r}, t) = \mathrm{j}\rho_0\omega \frac{\mathrm{e}^{\mathrm{j}\omega t - \mathrm{j}kr}}{r} [q_{20} Y_{20}(\theta, \phi) + q_{2-2} Y_{2-2}(\theta, \phi) + q_{22} Y_{22}(\theta, \phi)]$$

$$= -\mathrm{j}\rho_0\omega Q(kd)^2 \frac{\mathrm{e}^{\mathrm{j}\omega t - \mathrm{j}kr}}{4\pi r} \left[\frac{1}{3} - \sin^2\theta \sin^2\phi\right]. \quad (35)$$

The lateral quadrupole has following non-zero q_{lm}:

$$q_{21} = \frac{1}{15} Q(kd)^2 \sqrt{\frac{15}{8\pi}}, \qquad q_{2-1} = q_{21} \quad \text{(36a, b)}$$

which result in

$$p(\mathbf{r}, t) = \mathrm{j}\rho_0\omega \frac{\mathrm{e}^{\mathrm{j}\omega t - \mathrm{j}kr}}{r} [q_{2-1} Y_{2-1}(\theta, \phi) + q_{21} Y_{21}(\theta, \phi)]$$

$$= \rho_0\omega Q(kd)^2 \frac{\mathrm{e}^{\mathrm{j}\omega t - \mathrm{j}kr}}{4\pi r} \sin\theta \cos\theta \sin\phi. \quad (37)$$

The directivity patterns of the longitudinal and lateral quadrupole sound radiation are shown in Table 2.

3.4. SOUND RADIATION BY A TRIPOLE SOUND SOURCE

According to Huygens' principle [5], each point on a wavefront may be regraded as a source (Huygens' source) of secondary waves for the new wavefront at a later time. The

TABLE 2

Directivity distribution of a longitudinal and a lateral quadrupole sound radiation

Longitudinal quadrupole radiation $p(\mathbf{r}, t) = -\mathrm{j}\rho_0\omega Q(kd)^2 \frac{\mathrm{e}^{\mathrm{j}\omega t - \mathrm{j}kr}}{4\pi r} [\frac{1}{3} - \sin^2\theta \sin^2\phi]$	
Lateral quadrupole radiation $p(\mathbf{r}, t) = \rho_0\omega Q(kd)^2 \frac{\mathrm{e}^{\mathrm{j}\omega t - \mathrm{j}kr}}{4\pi r} \sin\theta \cos\theta \sin\phi$	

strength of the Huygens' source consists of a dipole term due to the pressure on the surface of the wavefront, and a monopole term due to the air particle velocity. Using the Helmholz–Kirchhoff integral for sound pressure in a space enclosed by the wavefront surface and the surface at the infinity, the radiated sound pressure by a Huygens' source at the wavefront $\mathbf{r}'$ away from a point source is [6]

$$p_H(r,\omega) = \frac{A'}{r}\,[(1+\mathrm{j}kr') + \mathrm{j}kr'\cos\theta]\,\mathrm{e}^{-\mathrm{j}kr}, \tag{38}$$

where r is the distance from Huygens' source to the observation point in the farfield ($kr \gg 1$), θ is an angle between $\mathbf{r}'$ and $\mathbf{r}$ and A' is proportional to the sound pressure at $\mathbf{r}'$. The sound radiation directivity can be interpreted as the superposition of sound field radiated by a dipole and a monopole mode witht he dipole vector direction perpendicular to that of $\mathbf{r}'$:

$$p_H(r,\omega) = \frac{\sqrt{4\pi}A'}{r}\left[(1+\mathrm{j}kr')\,Y_{00} + \frac{\mathrm{j}kr'}{\sqrt{3}}\,Y_{10}\right]\mathrm{e}^{-\mathrm{j}kr}. \tag{39}$$

Figure 4 shows the magnitude directivity of a Huygens' source for $kr' \gg 1$.

A tripole sound source has been realized by Tian and Sha [6] in a limited range of frequency. In their design, two loudspeakers were used. One of them within an enclosure was used as the monopole source and the other coupled with two short tubes of proper length in its front and rear surfaces was used as a dipole source. The tubes' diameter are the same as that of the loudspeakers.

4. SOUND POWER

The sound power radiated from a confined source source is calculated by the farfield sound pressure and air particle velocity v_r in the radius direction:

$$W = \frac{1}{2}\int_0^{\pi}\int_0^{2\pi} \mathrm{Re}[p v_r^*]\, r^2 \sin\theta\, \mathrm{d}\theta\, \mathrm{d}\phi. \tag{40}$$

In the far field,

$$v_r(\mathbf{r},t) = -\frac{1}{\mathrm{j}\rho_0\omega}\frac{\partial p(\mathbf{r},t)}{\partial r} \cong \frac{\mathrm{j}\rho_0\omega}{\rho_0 c_0}\frac{\mathrm{e}^{\mathrm{j}\omega t - \mathrm{j}kr}}{r}\sum_{l=0}^{\infty}\sum_{m=-l}^{l} q_{lm} Y_{lm}(\theta,\phi). \tag{41}$$

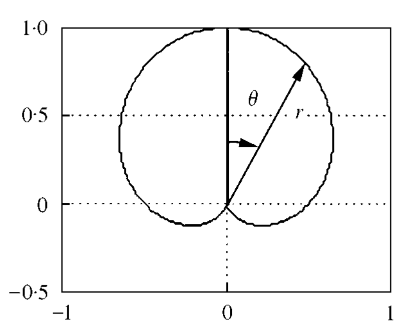

Figure 4. Magnitude directivity of a Huygens source.

The orthogonality of the spherical harmonic functions

$$\int_0^{\pi}\int_0^{2\pi} Y^*_{l'm'}(\theta,\phi)\,Y_{lm}(\theta,\phi)\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi = \delta_{l'l}\delta_{m'm} \tag{42}$$

allows the multipole radiating power to be expressed as

$$W = \frac{(\rho_0\omega)^2}{2\rho_0 c_0}\sum_{l=0}^{\infty}\frac{1}{(2l+1)^2}\sum_{m=-l}^{l}\left|\int q_v(\mathbf{r}_0)a_l Y^*_{lm}(\theta_0,\phi_0)\,\mathrm{d}V_0\right|^2. \tag{43}$$

If d denotes the characteristic dimension of the sound source, then $a_1 \cong jkd(r_0/d)$ and $a_2 \cong \frac{1}{3}(jkd)^2(r_0/d)^2$. From equation (43), the power radiated from monopole, dipole and quadrupole are related to kd as follows:

$$W_M = \frac{(\rho_0\omega)^2}{2\rho_0 c_0}\left|\int q_v(\mathbf{r}_0)\,Y^*_{00}(\theta_0,\phi_0)\,\mathrm{d}V_0\right|^2, \tag{44a}$$

$$W_D = \frac{(\rho_0\omega)^2}{18\rho_0 c_0}(kd)^2\sum_{m=-1}^{1}\left|\int q_v(\mathbf{r}_0)(r_0/d)\,Y^*_{1m}(\theta_0,\phi_0)\,\mathrm{d}V_0\right|^2 \tag{44b}$$

and

$$W_Q = \frac{(\rho_0\omega)^2}{450\rho_0 c_0}(kd)^4\sum_{m=-2}^{2}\left|\int q_v(\mathbf{r}_0)(r_0/d)^2\,Y^*_{2m}(\theta_0,\phi_0)\,\mathrm{d}V_0\right|^2. \tag{44c}$$

5. CONCLUSIONS

In this note, the radiating field from confined sound sources is described by radiating modes, which are mathematically orthonormal and representing the basic components of the radiation directivity. The description becomes possible because of the use of a novel expansion of the exponential term in equation (8) by Legendre polynomials (see equation (10)–(17)). Using this description, the radiating field from confined sound sources can be readily analyzed by the coefficients of the radiating modes q_{lm}. A general expression for multiple radiating power is also obtained.

REFERENCES

1. P. M. MORSE and K. U. INGARD 1986 *Theoretical Acoustics*. Princeton, NJ: Princeton University Press.
2. A. D. PIERCE 1981 *Acoustics: An Introduction to its Physical Principles and Applications*. New York: McGraw-Hill.
3. A. P. DOWLING 1996 *Encyclopedia of Acoustics*, (M. J. Crocker, editor), 107–125. New York: John Wiley & Sons, Inc. Steady-state radiation from sources.
4. J. D. JACKSON 1975 *Classical Electrodynamics*. New York: John Wiley & Sons, Inc.
5. S. P. PARKER 1983 *Encyclopedia of Physics*. USA: McGraw-Hill.
6. J. TIAN and J. Z. SHA 1987 *Inter-Noise*'87, 553–556. Secondary sources of a spatial active noise attenuation.